Mathematics I Midterm Exam

مجموعة ( )	سكشن ( )	اسم الطالب:

(1) By Mathematical Induction prove that $\sum_{r=1}^{n}\frac{1}{2^r}=1-\frac{1}{2^n}$ is true for any positive integer n.

Answer

The statement is true for n=1 .Let the statement is true for n=k i.e. $\sum_{r=1}^{n}\frac{1}{2^r}=1-\frac{1}{2^k}$

When n=k+1 $\sum_{r=1}^{k+1}\frac{1}{2^r}=\sum_{r=1}^{k}\frac{1}{2^r}+\frac{1}{2^{k+1}}=1-\frac{1}{2^k}+\frac{1}{2^{k+1}}=1-\frac{2}{2^{k+1}}+\frac{1}{2^{k+1}}=1-\frac{1}{2^{k+1}}$

The statement is true for n=k+1 then the statement is true for any positive number n.

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(2) Find the sum $\sum_{r=1}^{n}(2r-1)(2r+1)$

Answer

$$u_r=(2r-1)(2r+1)=\frac{(2r-1)(2r+1)(2r+3)}{6}-\frac{(2r-3)(2r-1)(2r+1)}{6}=f(r+1)-f(r)$$

$$S_n=\frac{(2n-1)(2n+1)(2n+3)}{6}+\frac{1}{2}$$

$$\sum_{r=1}^{n}(2r-1)(2r+1)=\frac{(2n-1)(2n+1)(2n+3)}{6}+\frac{1}{2}$$

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(3) Find coefficient x^{10} in the expansion $(1+x)^2(1-x)^{-4}$.

Answer

$$(1+x)^2(1-x)^{-4}=\left(1+2x+x^2\right)\sum C_r^{r+4-1}\ x^r==\left(1+2x+x^2\right)\sum_{r=0}^{\infty}C_r^{r+3}\ x^r$$

Coefficient x^{10} is $C_{10}^{13}+2C_9^{12}+C_8^{11}=C_3^{13}+2C_3^{12}+C_3^{11}$

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(4) Write the first four terms in the expansion $\sqrt[3]{8+x}$

Answer

$$\sqrt[3]{8+x}=2(1+\frac{x}{8})^{\frac{1}{3}}=2\left[1+\frac{1}{3}(\frac{x}{8})+\frac{1}{2!}\frac{1}{3}(\frac{1}{3}-1)(\frac{x}{8})^2+\frac{1}{3!}\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)(\frac{x}{8})^3+...\right]$$

$$=2\left[1+\frac{x}{24}-\frac{x^2}{576}+\frac{5x^3}{41472}+...\right]$$